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Simulation of Rotating Black Holes

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ABSTRACT

Considering the analogy between classical thermodynamic parameters and black hole parameters, the four laws of thermodynamics are reinterpreted for Kerr and Kerr-Newman black holes. A simple model for the dynamic relationships was obtained by considering the surface area of the outer horizon of a Kerr and Kerr-Neumann black hole as the area of a perfect black body. Finding the conditions these black holes should satisfy, the equations were numerically solved and simulated for the Hawking temperature, Hawking radiation and entropy variations of Kerr and Kerr-Newman black holes. The Hawking temperature and Hawking radiation of a given rotating black hole drastically increases at the final stage of the black hole evaporation. In the meantime, the entropy of a rotating black hole drastically decreases through the time. The additional angular momentum and the charge effects gain high Hawking temperatures and high Hawking radiation values for the black holes while these effects reduce the entropy of the black holes.

***Keywords*:** Kerr black hole, Kerr-Newman black hole, rotating black holes, Hawking radiation, Hawking temperature, thermodynamic parameters, mass variation, entropy

1. INTRODUCTION

Black holes are among the most fascinating things in the universe. They results from a competition between nuclear fusion reaction inside the core and the gravitational collapsing of a star. Eventually the fuel of the star will be totally consumed leaving only the gravitational collapsing of the star leading to the birth of a black hole through a supernova explosion. The

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region inside the horizon, once the star has shrunk away to nothing is empty and from the exterior universe, black and inaccessible. It is therefore called a black hole. Only stars above the 3 solar masses will form a singularity with infinite dense and end up as black holes [1].

It was discovered in 1963 that an exact spacetime exists for a black hole with just mass and angular momentum known as Kerr geometry, and in 1965 a solution including charge was found for Kerr-Newman geometry. A black hole is immensely dense and its escape velocity is even higher than the velocity of light. The boundary at where the escape velocity equals the velocity of light is the event horizon, a hypothetical sphere or an ellipsoid depending on whether the black hole is having an angular momentum or not. Beyond the event horizon of a black hole all the known physical laws are no longer valid, and one needs a quantum theory of gravity to seek in to the ultimate singularity at the centre of the black hole. The key to understanding black holes is to appreciate the meaning of the so-called event horizon [2]. Unlike steady black holes, rotating black holes have complex structure consisting with an outer horizon, inner horizon and a region between static limit and the horizon known as the ergosphere (Fig. 1).

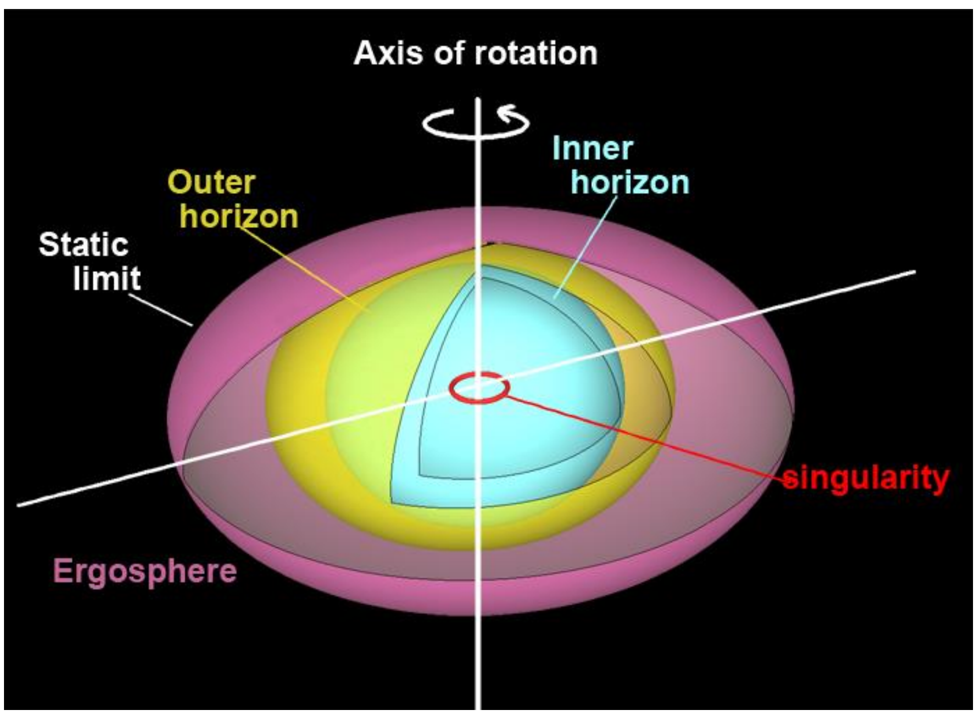

Figure 1. Structure of a rotating black hole.

Classically black holes were nature's ultimate sponges, absorbing all matter and emitting nothing. Superficially they had neither temperature nor entropy and were characterized by only a few basic parameters: mass, angular momentum, and charge [3]. The advent of quantum field theory in curved space-time changed all of these, leading to the area of a black hole corresponding to its entropy [4] and its surface gravity corresponding to its temperature [5]. All theoretical evidence indicated that black holes radiate heat, analogous to black body radiation. The laws of gravitation were posited to be intimately connected with the laws of thermodynamics [6, 7]. The Hawking's area theorem [8], states that the area of the event horizon

of a black hole can never decrease. Bekenstein [4] subsequently noticing the resemblance between this area law and the second law of thermodynamics, applied thermodynamic considerations in a set of thought experiments and proposed that each black hole should be assigned an entropy proportional to the area of its event horizon. Pursuing this analogy further, the four laws of black hole mechanics were formulated by Bardeen, Carter, and Hawking [9] under the assumption that the event horizon of the black hole is a Killing horizon, which is a null hypersurface generated by a corresponding Killing vector field. By treating the space time curvature at the event horizon of a black hole through quantum field theory, Hawking [10] showed that the black hole should emit particles with a characteristic black body spectrum of a temperature which was later named as Hawking temperature.

In this paper by considering the surface area of the outer horizon of a Kerr and Kerr-Neumann black hole as the area of a perfect black body, a simple model was obtained for the dynamic relationships. Equations for the entropy, Hawking temperature and Hawking radiation for Kerr and Kerr-Neumann black holes were derived. Finding the conditions these black holes should satisfy; the equations were numerically solved and simulated. The temperature and Hawking radiation of Kerr and Kerr-Neumann black hole drastically increases at the final stage of the black hole evaporation while the entropy drastically decreases with the time. The angular momentum and the charge effects increase the temperature and Hawking radiation.

2. THEORY OF ROTATING AND CHARGED BLACK HOLES

The search for axially symmetric solutions of the Einstein field equation started in 1917 with static and was extended in 1924 to stationary metrics. It culminated in 1963 with the discovery of the Kerr metric. Accordingly to the No-Hair theorem all black holes solutions of the Einstein-Maxwell equation of electromagnetism in general relativity can be completely characterized by their observable classical parameters mass, electric charge and angular momentum. An exact solution of Einstein field equation in general relativity is the Kerr metric or Kerr geometry that describes the geometry of empty space-time around a rotating uncharged axially-symmetric black hole with a quasi-spherical event horizon. The geometry of space-time for a rotating charged black hole with mass M, charge Q and angular momentum J is described by Kerr–Newman metric. The charged black holes are irrelevant as they attract other charged particles and become neutralised. The classifications of black holes are tabulated in Table 1 [11].

The most general solution of a charged, spinning black hole is given by the Kerr-Newman metric. One way to express it is by writing down its line element in a particular set of spherical coordinates, also called Boyer–Lindquist coordinates [12]:

$$ds^2 = \frac{-\Delta}{\rho^2}\left(dt - a\,\sin^2\theta d\phi\right)^2 + \frac{\sin^2\theta}{\rho^2}\left[\left(r^2+a^2\right)d\phi - a\,dt\right]^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 \tag{1}$$

where:

$$\Delta = r^2 - 2Mr + a^2 + Q^2, \qquad \rho^2 = r^2 + a^2\cos^2\theta \qquad \text{and} \qquad a = \frac{J}{Mc}$$

a is the Kerr parameter with unit length and c is the velocity of light in vacuum. Though electric charge is clearly independent of the other two parameters mass and angular momentum are not independent. By introducing the Kerr parameter the angular momentum is considered independent from the mass. Therefore, the three parameters could be treated independently and the calculations are easy to carry out.

Table 1. Classifications of black holes.

Type of Black hole	Properties	Parameters
Schwarzschild	Steady and spherical	Mass $(Q=0, J=0)$
Kerr	Rotation	Mass and angular momentum $(Q=0, J>0)$ (Astrophysically relevant)
Kerr-Newman	Rotation and charged	Mass, angular momentum and electric charge $(Q\neq 0, J>0)$
Reissner–Nordström	Steady and charged	Mass and electric charge $(Q\neq 0, J=0)$ (Not astrophysically relevant)

The condition for an event horizon is given where the $g_{rr}=0$ term in the metric is singular. This is when the denominator of that term goes to zero and then gives values of the radii from a quadratic equation. The solutions are the inner r_- and an outer horizon r_+ given by [13]:

$$r_\pm = \frac{G}{c^2}\left[M \pm \left(M^2 - \frac{a^2c^4}{G^2} - \frac{1}{4\pi\varepsilon_0 G}Q^2\right)^{1/2}\right] = M \pm \sqrt{M^2 - a^2 - Q^2} \tag{2}$$

The second solution is given in terms Planck units ($\hbar = G = c = k = 1/4\pi\varepsilon_0 = 1$) where $a = J/M$, so that everything is in terms of mass. The solution r_+ also denoted by R_H is usually associated with the outer event horizon.

For the Kerr-Newman metrics which are the unique asymptotically flat stationary black holes in Einstein-Maxwell theory, one can get explicit expressions for the area A, surface gravity κ, angular velocity Ω and electrostatic potential Φ of the black hole horizon in terms of the macroscopic conserved quantities using the values M, angular momentum $J = aM$ and

charge Q. The area A of a rotating black hole, the angular velocity Ω and surface gravity κ can be shown as [14]:

$$A = 4\pi(r_+^2 + a^2) = 4\pi\left[2\left(\frac{GM}{c^2}\right)^2 - \frac{GQ^2}{4\pi\varepsilon_0 c^4} + 2\left(\frac{GM}{c^2}\right)^2\sqrt{1 - \frac{Q^2}{4\pi\varepsilon_0 GM^2} - \frac{a^2c^4}{G^2M^2}}\right] \quad (3)$$

$$\Omega = \frac{4\pi a}{A} = \frac{a}{(r_+^2 + a^2)}; \qquad \kappa = \frac{4\pi(r_+ - M)}{A} \quad (4)$$

The electromagnetic potential Φ and the non-vanishing components of the electromagnetic vector potential A_α are given by:

$$\Phi = \frac{4\pi Q r_+}{A} = \frac{Q r_+}{(r_+^2 + a^2)}; \qquad \mathrm{A}_\alpha = -\frac{Qr}{\rho}(dt - a\sin^2\theta d\phi) \quad (5)$$

There is a close relationship between the traditional thermodynamic parameters and black hole parameters; the internal energy E to mass M of the black hole, temperature T to the surface gravity κ of the event horizon and the entropy S to the area A of the event horizon. The zeroth, first, second and third laws of classical thermodynamics can be re-interpreted for a black hole [4]. If area A of the event horizon plays the role of entropy, then surface gravity κ plays the role of temperature. Surface gravity defined locally, is always constant over the event horizon of a stationary black hole.

This constancy is reminiscent of the zeroth law of thermodynamics which states that the temperature is uniform everywhere in a system in thermal equilibrium. Considering a quasi-static process in which a small mass is added to a black hole, the increase of charge Q and angular momentum J by dQ and dJ respectively comes at the price of increasing M by dM and thus area A by a certain amount through the event horizon r_+. Therefore the first law of black hole mechanics essentially the same as the first law of thermodynamics and expresses the conservation of energy in the following way:

$$dM = \frac{\kappa}{8\pi}dA + \Omega dJ + \Phi dQ \quad (6)$$

Ω and Φ are the angular velocity and the electrostatic potential of the outer horizon respectively. An extended form of the zeroth law implies that not only the surface gravity κ, but also the angular velocity Ω and the electrostatic potential Φ are constant over the event horizon of any stationary black hole.

The second law of black hole mechanics is Hawking's area theorem that is the area A of a black hole horizon cannot decrease $(A \geq 0)$. This is obviously analogous to the second law of thermodynamics, that the entropy S of a closed system cannot decrease. Cosmic censorship implies the unattainability of absolute zero, $\kappa = 0$. It is straightforward to show that if J^2 or Q^2 become large enough such that [15]:

$$\frac{J^2}{M^4}+\frac{Q^2}{M^2} = 1 \tag{7}$$

then κ vanishes although A does not. A black hole with such parameters is known as an extreme Kerr-Newman black hole. It is the limiting case of an object still possesses an event horizon. Should the left-hand side become even infinitesimally greater than one, then the horizon would disappear and we would be left with a naked singularity, i.e. the singularity would no longer be invisible inside a black hole but would be able to influence, and be observed by the outside universe.

This circumstance is considered so undesirable for physics that most physicists believe in the so-called cosmic censorship hypothesis that naked singularities cannot form from gravitational collapse. Cosmic censorship implies the unattainability of 'absolute zero', $\kappa = 0$. This is the third law which states that the surface gravity of the horizon cannot be reduced to zero in a finite number of steps [15].

A classical black hole is a non-quantum black hole which emits neither matter nor radiation. But when the thermodynamic analogy is taken into account, a few oversights of the black hole thermodynamics became apparent: the temperature of a black hole vanishes; in natural units, the entropy is dimensionless whereas that of the event horizon, the area is length squared; the event horizon area of each black hole individually is non-decreasing whereas only the total entropy is non-decreasing in thermodynamics. To overcome these, a quantum black hole was introduced. Hawking radiation describes the radiation of black holes due to quantum effects around the event horizon [16].

Vacuum fluctuations and due to the uncertainty principle of energy and time constantly form particles, anti-particles and annihilate in the free space. Around the event horizon anti-particle can fall into the black hole and tunnelled through the universe observed as radiation. This radiation is known as the Hawking radiation. As a result of a falling anti-particle into the black hole it can be annihilated with a particle.

Hawking showed that the radiation emitted by black holes is due to thermal effects and a wavelength spectrum associated with it leads to the black body spectrum [5]. Applying quantum field theory to the curved spacetime of a black hole he found that the black hole emits electromagnetic radiation with a temperature called Hawking temperature T_H :

$$T_H = \frac{\hbar c(r_+ - r_-)}{4\pi k_B (r_+^2 + a^2)} = \frac{\hbar c}{4\pi k_B}\frac{\left(1-\frac{Q^2}{4\pi\varepsilon_0 GM^2}-\frac{a^2c^4}{G^2M^2}\right)^{\frac{1}{2}}}{\left(\frac{GM}{c^2}\right)\left[1+\left(1-\frac{Q^2}{4\pi\varepsilon_0 GM^2}-\frac{a^2c^4}{G^2M^2}\right)^{\frac{1}{2}}\right]-\frac{G}{4\pi\varepsilon_0 c^4}Q^2} \tag{9}$$

where: $\hbar$ is the reduced Planck constant and k_B is the Boltzmann constant. Combining the Stephan-Boltzmann law and Hawking's temperature formula, an expression for Hawking radiation can be derived:

$$P = \frac{\hbar c^6}{240\pi G^2} \frac{\left[1 - \frac{Q^2}{4\pi\varepsilon_0 GM^2} - \frac{a^2c^4}{G^2M^2}\right]^2}{M^2\left[2 + 2\left(1 - \left(\frac{ac^2}{MG}\right)^2 - \frac{1}{4\pi\varepsilon_0 GM^2}Q^2\right)^{\frac{1}{2}} - \frac{1}{4\pi\varepsilon_0 GM}Q^2\right]^3} \tag{10}$$

The entropy of a black hole called Bekenstein-Hawking entropy is proportional to the surface area A of its event horizon and is given by the following formula [13]:

$$S_{BH} = \frac{k_B c^3 A}{4G\hbar} = \frac{\pi k_B c^3}{\hbar G}\left[2\left(\frac{GM}{c^2}\right)^2 - \frac{G}{4\pi\varepsilon_0 c^4}Q^2 + 2\left(\frac{GM}{c^2}\right)^2\sqrt{1 - \frac{Q^2}{4\pi\varepsilon_0 GM^2} - \frac{a^2c^4}{G^2M^2}}\right] \tag{11}$$

where: M is the reduction mass $M(t)$. Due to the emission of Hawking radiation, the mass of a black hole always decreases with time.

The reduction mass parameter as a function of time is [17]:

$$M(t) = M_0{}^3 - \frac{\hbar c^4}{5120\pi G^2}t \tag{12}$$

where: M_0 is the initial mass of the black hole.

3. KERR-NEWMAN BLACK HOLE

The Kerr–Newman geometry solution has not been especially useful for describing astrophysical phenomena, because observed astronomical objects do not possess an appreciable net electric charge. The solution instead is of primarily theoretical and mathematical interest. As area of a Kerr-Newman black hole given in Eq. 3 should be real, the critical condition for Kerr-Newman black holes can be found as an inequality:

$$1 - \frac{Q^2}{4\pi\varepsilon_0 GM^2} - \frac{a^2c^4}{G^2M^2} \geq 0 \tag{13}$$

The critical surface is:

$$\frac{Q^2}{4\pi\varepsilon_0 GM_S^2} + \frac{a^2c^4}{G^2M_S^2} = \left(\frac{M}{M_S}\right)^2 \tag{14}$$

where: M_s is the mass of the sun. For different black holes mass to sun's mass ratios M / M_s Kerr parameters a and charge Q, were calculated for the critical value. The simulation is presented in Fig. 2. Below this critical surface all Kerr-Newman black holes are unphysical.

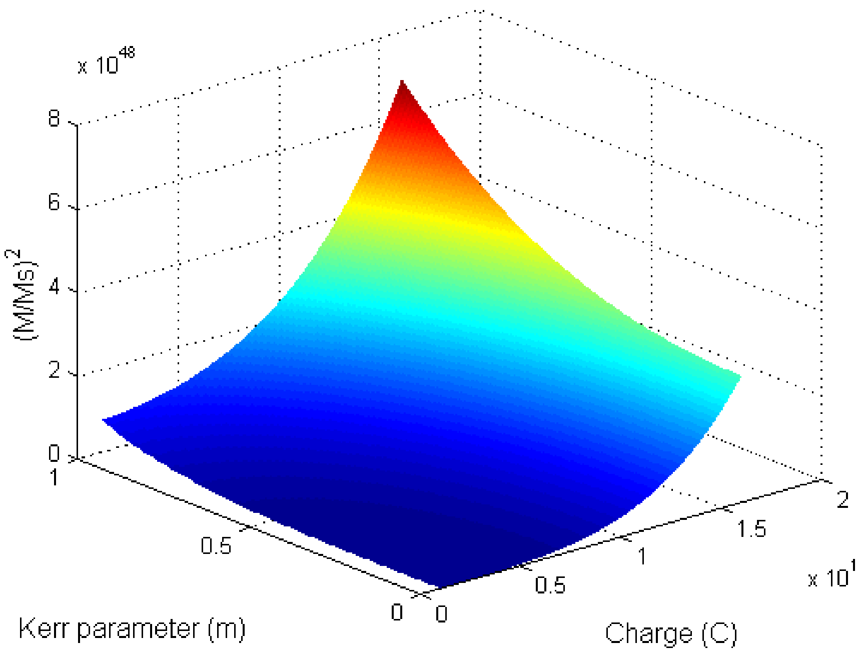

Figure 2. Critical limits for masses M/M$_s$ and Kerr parameter *a* and charge *Q* for Kerr-Newman black holes

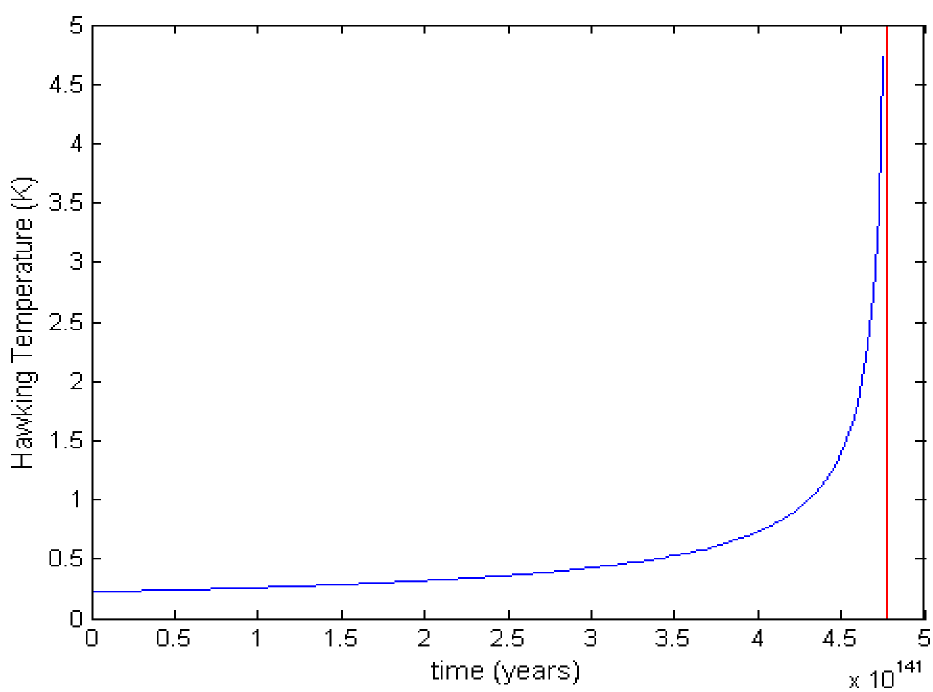

Figure 3. Hawking temperature variation against time

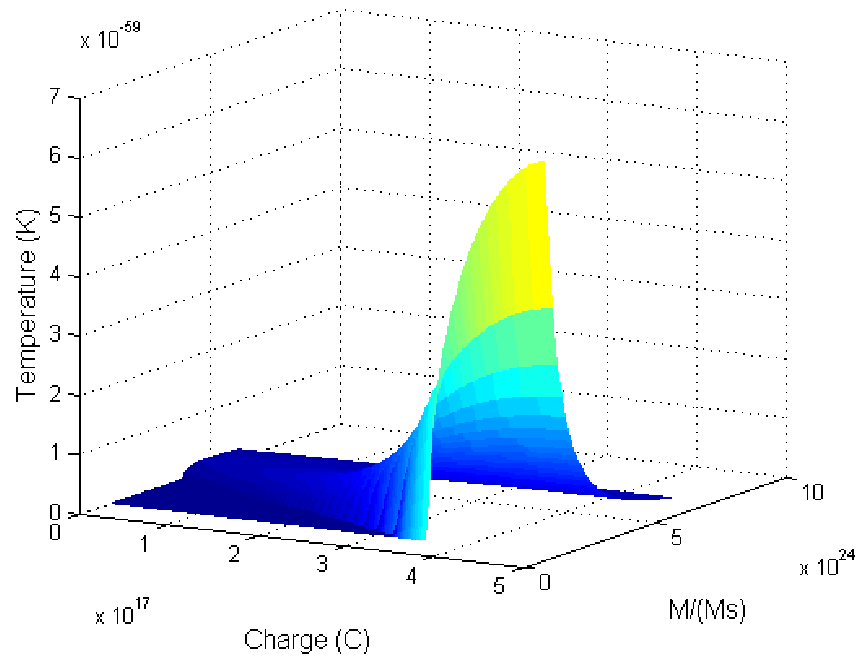

Figure 4(*a*). Hawking temperature variation over mass and charge of Kerr-Newman black holes

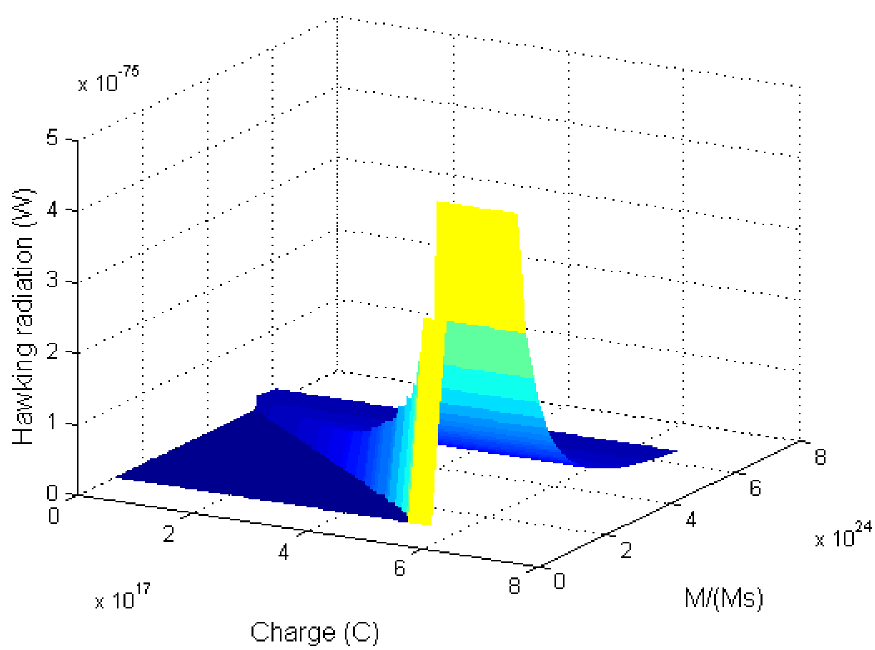

Figure 4(*b*). Hawking radiation variation over mass and charge of Kerr-Newman black holes

The Hawking temperature and the Hawking radiation given in Eq. 9 and Eq. 10 respectively were simulated for different Kerr parameters and charges of Kerr-Newman black

holes above the critical limits. The Hawking temperature variation against time for a period of $0 - 5 \times 10^{141}$ years for a particular Kerr-Newman black hole with $M / M_s = 6.09 \times 10^{24}$, $a = 0.5\text{m}$ and $Q = 1 \times 10^{17} C$ is presented in Fig. 3. Red line denotes the vanishing time of the black hole. Fig. 4(a) and Fig. 4(b) show the Hawking temperature and Hawking radiation variation against M / M_s and electric charge of Kerr-Newman black holes above the critical value. According to the Fig. 4(a) the increase in charge of a Kerr-Newman black hole increases the black hole temperature while higher masses decrease the temperature. The peak of the curve is at $M / M_s = 1.121 \times 10^{24}$, $Q = 4.9 \times 10^{17}\text{C}$ and Temperature $T = 6.64 \times 10^{-59} K$.

Fig. 4(b) shows that a heavily charged Kerr-Newman black hole emits more Hawking radiation than a less charged Kerr-Newman black hole, as well as a massive mass black hole emits lower radiation than a lower mass black hole.

The Bekenstein-Hawking entropy values for a Kerr-Newman black hole above the critical value were obtained from Eq. 13. Entropy variation over time for a different black hole masses are presented in Fig. 5(a). Red line denotes the epoch of the present time. The graphs show that, a massive black hole has more entropy than a light black hole at a given time. For a Kerr-Newman black hole of $M / M_s = 40.2$, the entropy variation with charge and Kerr parameter are presented in Fig. 5(b). This shows that the entropy decreases when the charge or the Kerr parameter is increased. This variation can be used to predict the stability of Kerr-Newman black holes. When a charge or Kerr parameter value is added to a given Kerr-Newman black hole the stability of that black hole decreases.

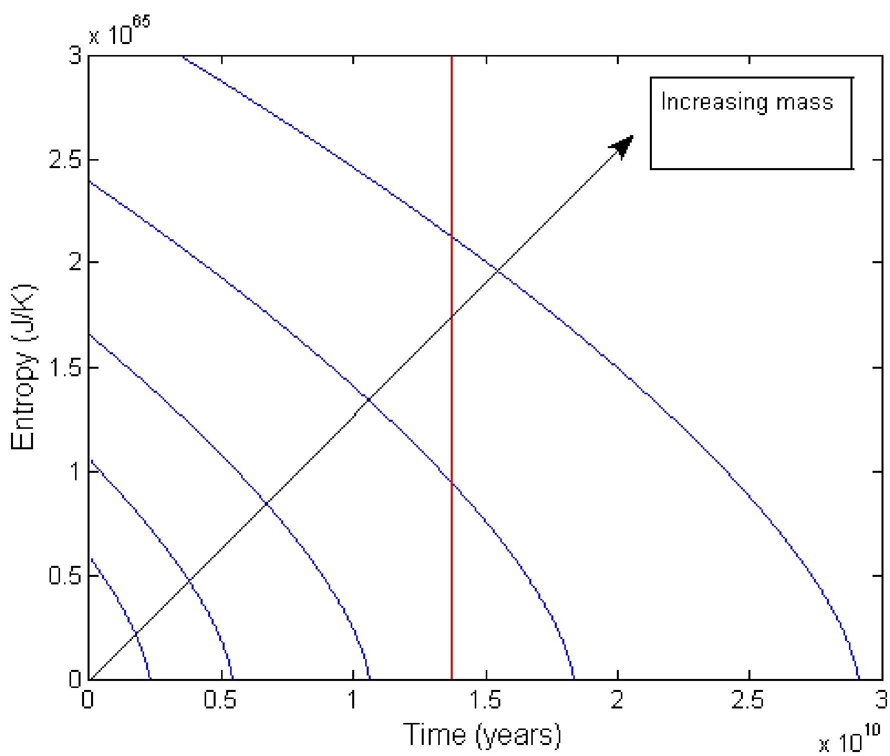

Figure 5(a). Entropy variation over time for Kerr-Newman black holes

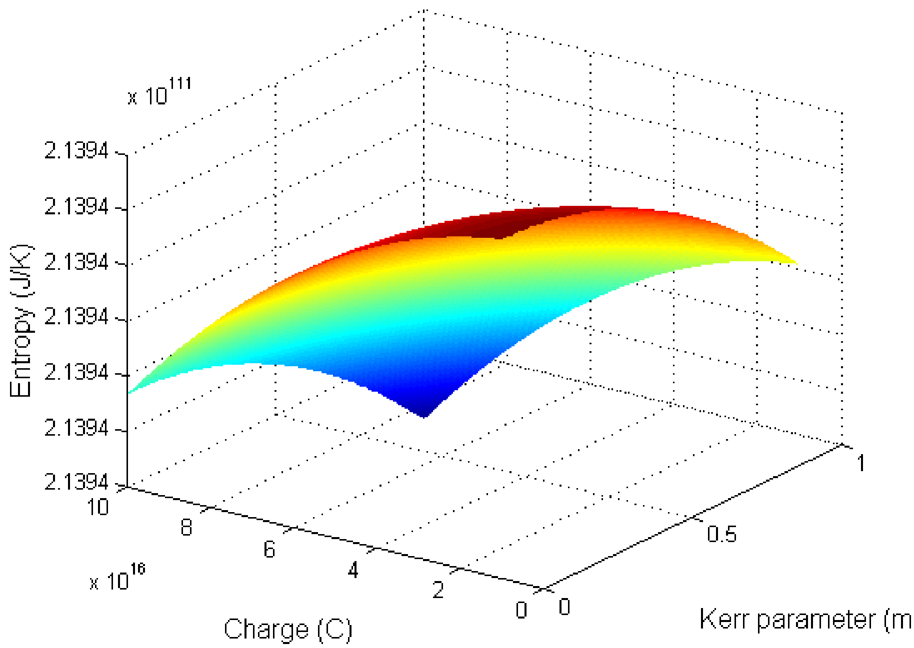

Figure 5(b). Entropy variation over Kerr parameter and charge for Kerr-Newman black hole of $M/M_s = 40.2$

4. KERR BLACK HOLE

Kerr black holes only possess a rotation and therefore characterized by its mass and the Kerr parameter $a = J/Mc$. Kerr geometry has two obvious symmetries, the time translations generated by ∂_t , and rotations around ϕ generated by ∂_ϕ. They have corresponding conserved quantities, mass M and angular momentum J. In addition to the Killing symmetries, the Kerr solution also has discrete symmetries. It is invariant under $(t,\phi) \to (-t,-\phi)$. This is consistent with the understanding that the Kerr solution represents a rotating black hole, so that time inversion also inverts the angular velocity [12]. The area of a Kerr black hole from Eq. 3 reads:

$$A = 8\pi\left(\frac{GM}{c^2}\right)^2\left[1+\sqrt{1-\frac{a^2c^4}{G^2M^2}}\right] \tag{15}$$

A black hole that violates the condition $X = \left(1-\dfrac{a^2c^4}{G^2M^2}\right) \le 0$, is an unphysical black hole as it gives an imaginary area. For different black hole M/M_s values and different Kerr parameters a , the X values were calculated and presented in Fig. 6. The black surface is the critical $X = 0$ plane. Below this plane all Kerr black holes with $M/M_s < 0.5\times10^{27}$ values and corresponding Kerr parameter cannot exist in the universe.

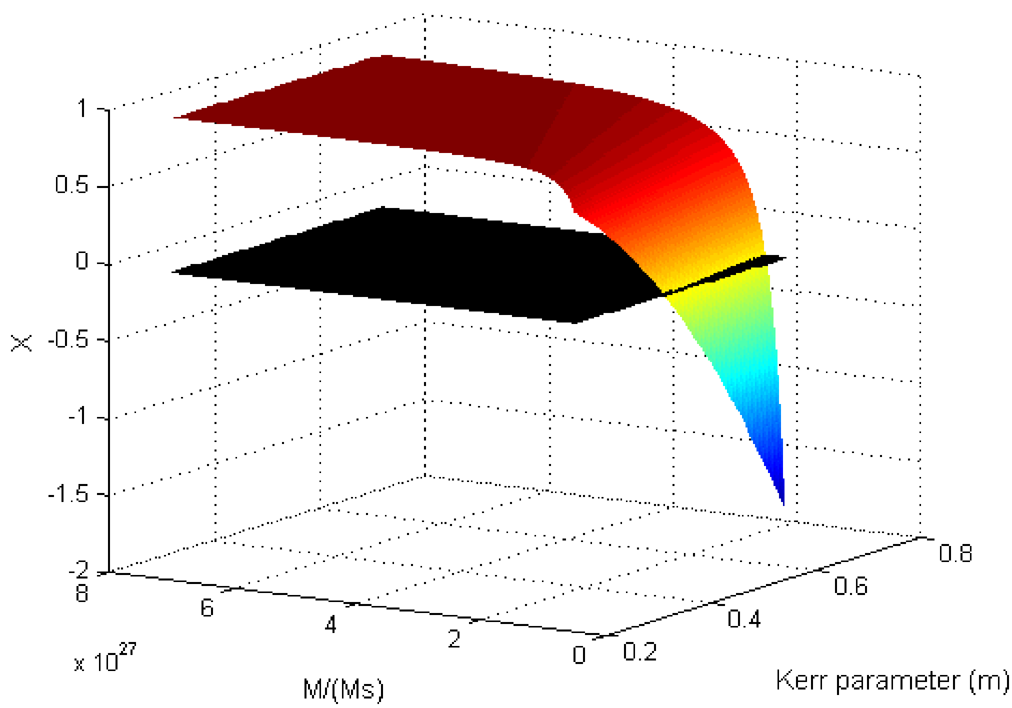

Figure 6. Critical masses M/M$_s$ and Kerr parameter values for Kerr black holes

The Hawking temperature and the Hawking radiation for Kerr black hole can be obtained from Eq. 9 and Eq.10 respectively:

$$T_H = \frac{\hbar c^3}{4\pi k_B GM} \frac{\left(1 - a^2c^4 / G^2M^2\right)^{\frac{1}{2}}}{\left[1 + \left(1 - a^2c^4 / G^2M^2\right)^{\frac{1}{2}}\right]} \tag{16}$$

$$P = \frac{\hbar c^6}{240\pi G^2 M^2} \frac{\left[1 - \left(ac^2 / GM\right)\right]^2}{\left[2 + 2\left(1 - \left(ac^2 / GM\right)^2\right)^{\frac{1}{2}}\right]} \tag{17}$$

These were calculated considering the critical condition for Kerr black holes with different M / M_s values and different Kerr parameters a and presented in Fig. 7(a) and Fig. 7(b) respectively.

Hawking temperature decreases with the black hole mass ratio and increasing with Kerr parameter. Even though for lower mass ratio stellar black holes, Kerr parameter has some effect on the Hawking temperature, when it comes to intermediate black holes or super-massive black holes, Kerr parameter does not play any role and the mass ratio dominates the Hawking temperature.

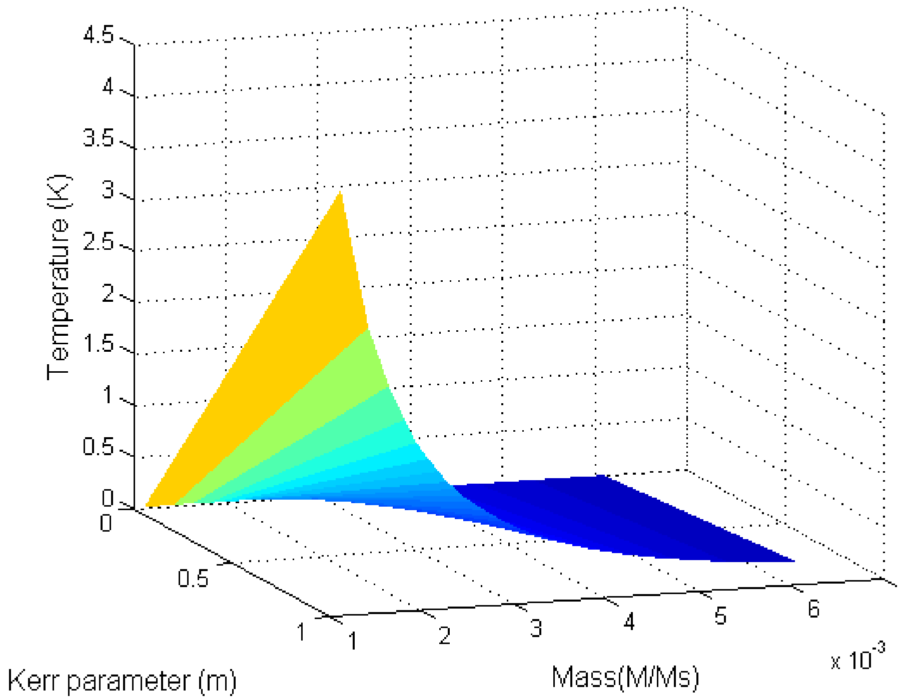

Figure 7(*a*). Hawking temperature with M/M$_s$ and Kerr parameter for Kerr black holes

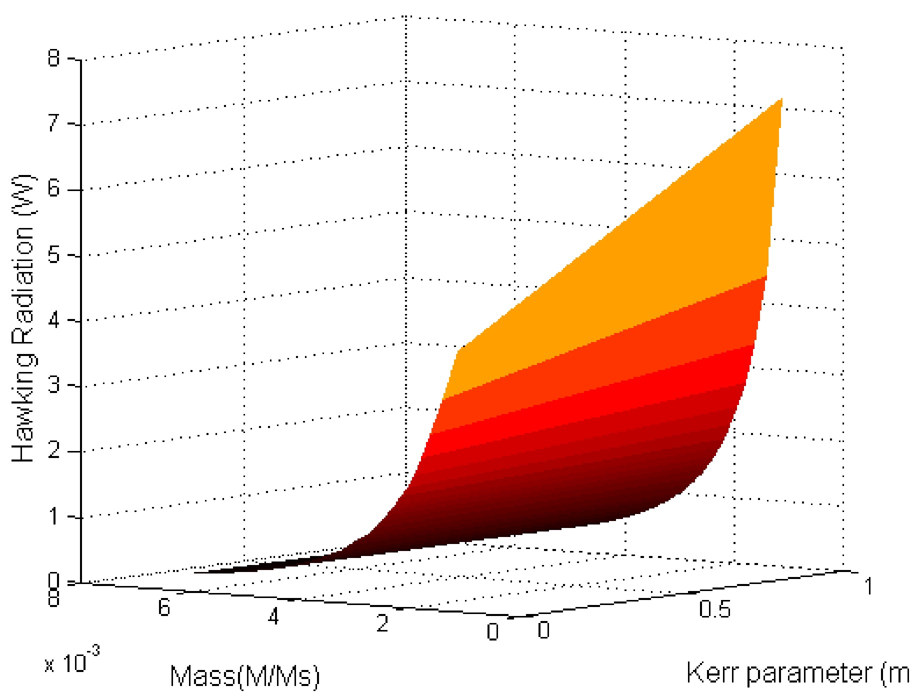

Figure 7(*b*). Hawking radiation with M/M$_s$ and Kerr parameter for Kerr black holes

For a Kerr black hole of $M / M_s = 0.0061$ and $a = 0.5\,\text{m}$, the Hawking temperature and radiation with time were simulated. The radiation variation with time is presented in Fig. 8. The red lines denote the time taken for vanishing of the black hole. In the final stage of the black hole, the temperature and radiation increase drastically within a short period of time and produces an enormous amount of Hawking temperature and radiation.

An expression for Kerr black hole entropy can be deduced from Eq. 11:

$$S = \frac{2\pi k_B G M^2}{\hbar c}\left(1+\sqrt{1-\frac{a^2c^4}{G^2M^2}}\right) \tag{18}$$

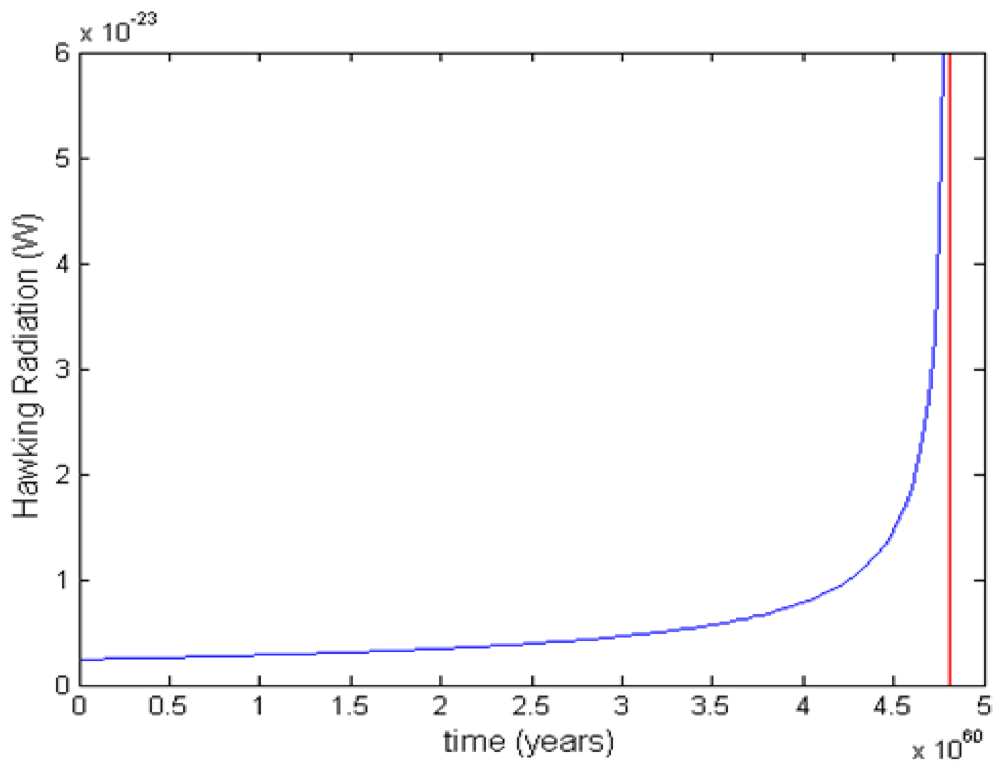

Figure 8. Hawking radiation with time for Kerr black hole of $M / M_s = 0.0061$ and $a = 0.5\,\text{m}$

Entropy variation for different Kerr black holes for $a = 0.2\,\text{m}$ with time is shown in Fig. 9. The red line denotes the current age of the universe which is 1.37×10^{10} years. At a given time a massive black hole has more entropy than a lighter black hole with the same Kerr parameter $a = 0.2\,\text{m}$. Black holes with larger masses initially possess a huge amount of entropy and with time, as the mass decreases, their entropy also decreased. This means that initially supermassive black holes have massive amount of entropy when compared to micro/mini black holes. Entropy variation of a Kerr black hole with $M / M_s = 3.52\times10^{-19}$ over time and Kerr parameter is presented in Fig. 11. Lower Kerr parameter black holes show higher stability than higher Kerr parameter black holes.

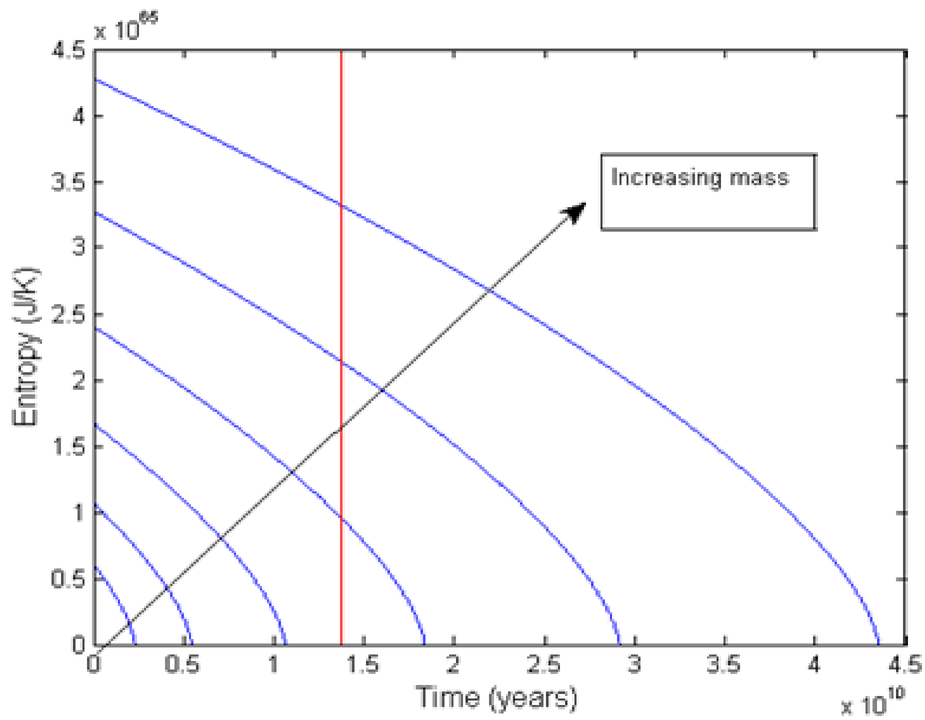

Figure 9. Entropy with time for Kerr black hole of $a = 0.2$m

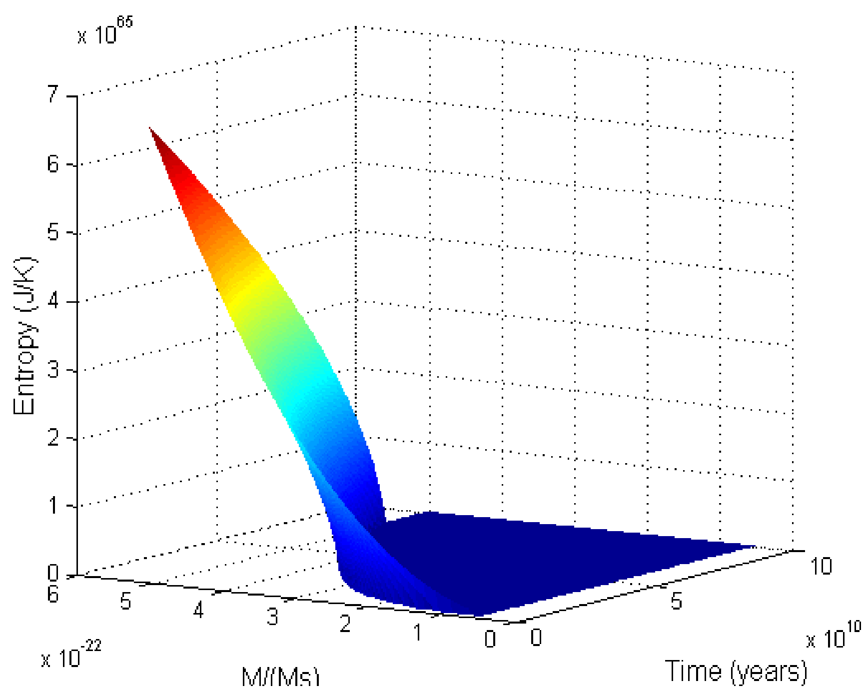

Figure 10. Entropy variation over time and mass for Kerr black hole of $a = 0.2$m

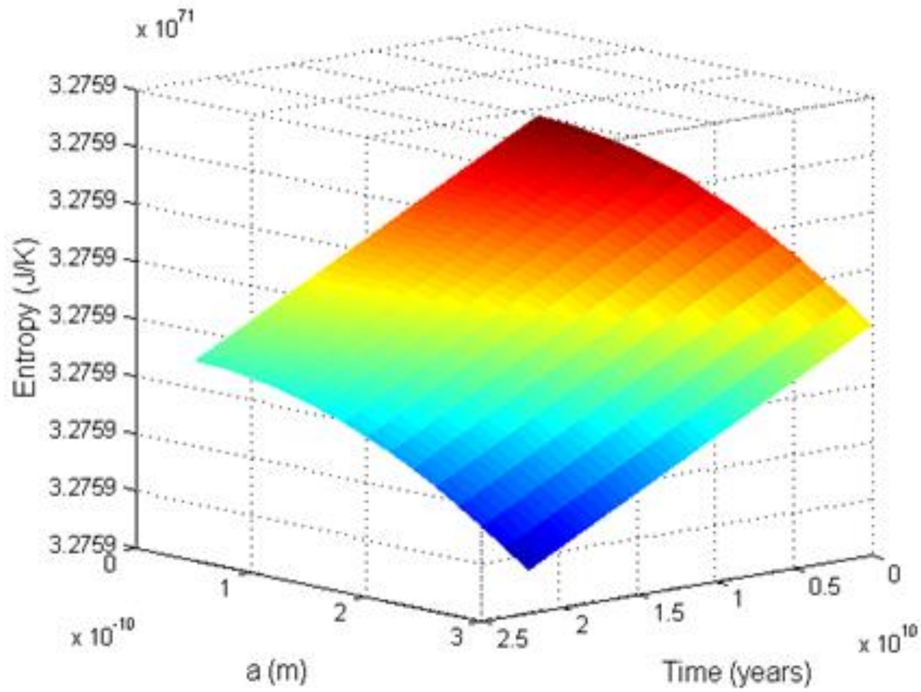

Figure 11. Entropy variation over time and Kerr parameter of a Kerr black hole

The Hawking radiation with respect to mass with Kerr parameter fixed at $a = 0.3\text{m}$ and entropy with time for Schwarzschild [18], Kerr and Kerr-Newman black holes are presented in Fig. 12 and Fig. 13 respectively. For a given mass value the Kerr-Newman black hole emits more radiation than the Kerr and Schwarzschild [18] black holes. But, when these black holes reach extreme masses, all of them tend to radiate similar amounts. Schwarzschild and Kerr black holes emit almost the same amount of radiation at extreme masses.

The rotating black holes emit more radiation than Schwarzschild black holes. For a given mass value a Kerr-Newman black hole has a higher temperature than Kerr and Schwarzschild black holes and when it comes to extreme mass conditions, temperature behaves in the same way as radiation. This implies that, at extreme mass conditions 'mass' parameter dominates the other parameters. In the entropy simulations (Fig.13), the entropy of the black holes decrease throughout the time and one could think of this as a violation of the second law of thermodynamics. Even though the entropy of a black hole is reduced, at the same time that part of the entropy is carried away to the universe by Hawking radiation.

Therefore, generally the second law of thermodynamics can be presented as the summation of the entropies of the black hole (S_{Bh}) and the universe $(S_{universe})$ always increases with time $(S_{Bh} + S_{universe} \geq 0)$. The Hawking radiation for any type of black hole with temperature $10^{-7}\,\text{K}$ was simulated using Stephan-Boltzmann equation at the outer event horizon. This is shown in Fig. 14. The red line denotes peak of the radiation jet of the black hole. This is exactly at the centre of the black hole. Hawking radiation distribution on the surface of the black hole from the top view is presented in Fig. 15. The peak of the radiation is at the centre of the black hole and decays gradually towards the event horizon.

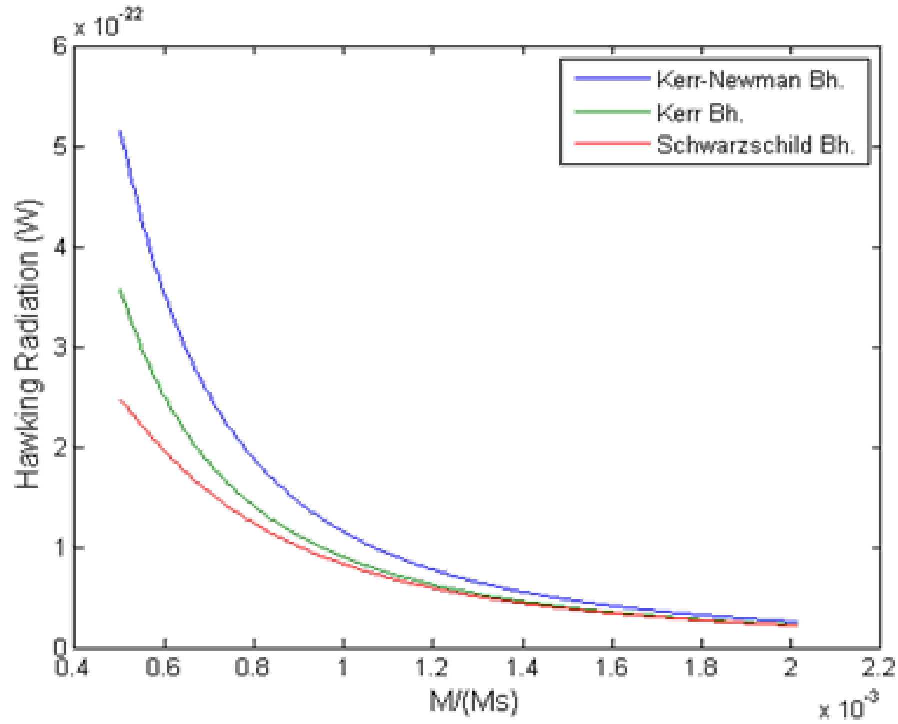

Figure 12. Hawking radiation over mass for Schwarzschild, Kerr and Kerr-Newman black holes $a = 0.3\text{m}$

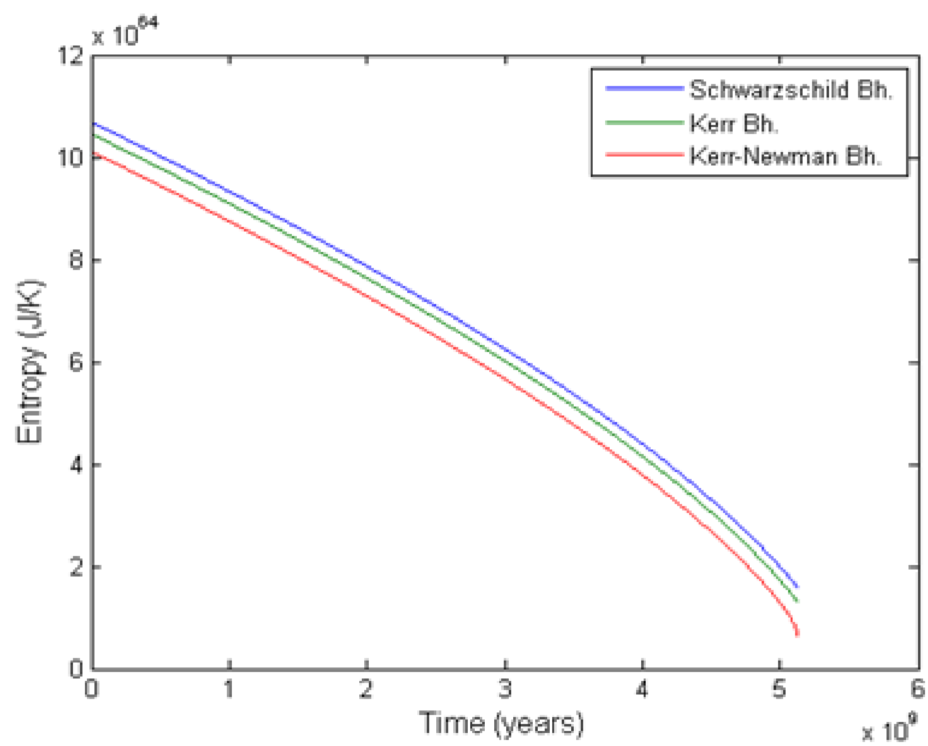

Figure 13. Entropy with time for Schwarzschild, Kerr and Kerr-Newman black holes

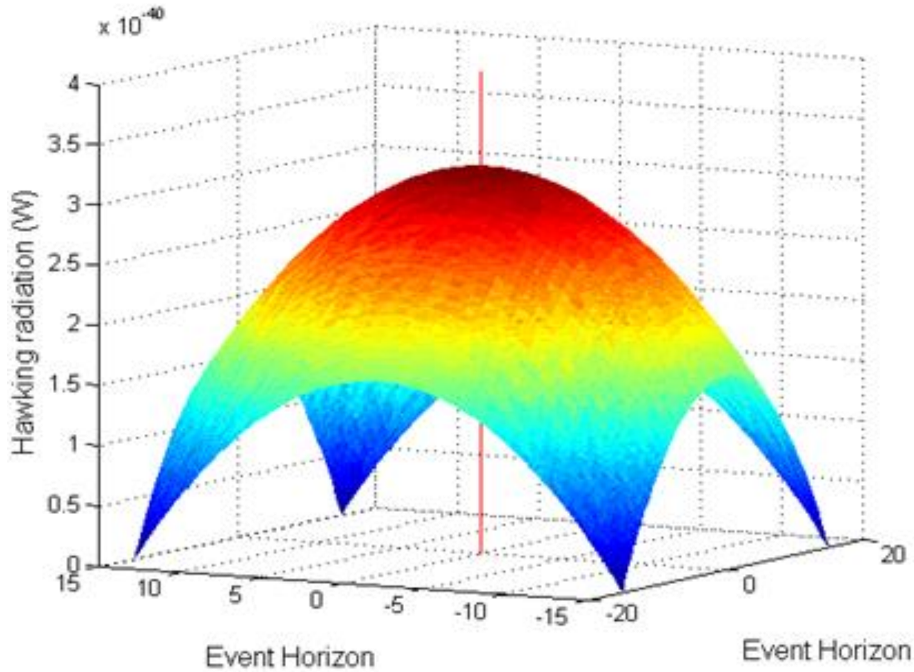

Figure 14. Hawking radiation at the event horizon of a black hole.

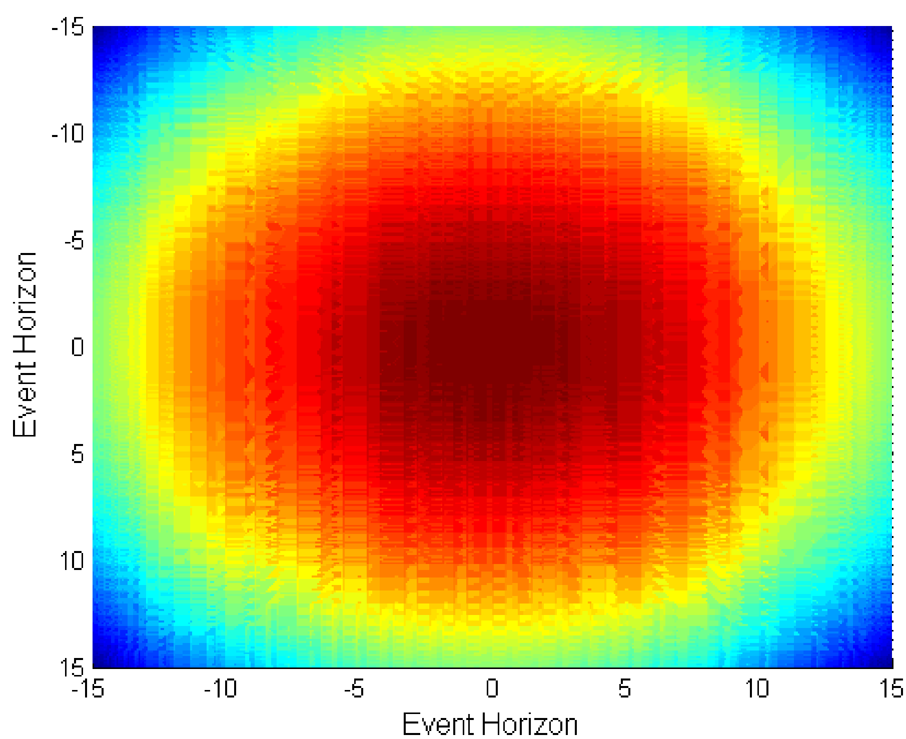

Figure 15. Top view of Hawking radiation at the surface of a black hole.

5. CONCLUSIONS

By extending the concepts of thermodynamics to include rotating black holes, the uncharged Kerr and charged Kerr-Newman black hole regions, the thermodynamics laws may be generalized with remarkable ease to encompass these situations. Kerr and Kerr-Newman solutions for Einstein's field equation demand that not only the mass, angular momentum and the electric charge can be considered as black hole parameters but there are some limiting parametric conditions of achieving rotating black hole states. Depending on these conditions rotating black holes can be physical or unphysical in the universe. The mathematical model assume that the energy is simultaneously gained and lost by the black hole according to the Stephan-Boltzmann law.

The simulations were done under the condition that area of a black holes must be real. All the temperature variations and radiation variations over the time for Kerr black holes and Kerr-Newman black holes show that, at the vanishing time of the black hole, it produces an infinite amount of temperature and radiation power within a short period of time. This suggests that the black hole evaporations could lead to violent explosions in the universe such as gamma ray bursts. Heavily charged and larger Kerr parameter black holes create higher temperature values and emit higher radiations. Also, when the mass of black holes is of intermediate or supermassive levels, the temperature and Hawking radiation drastically decrease and 'mass' parameter dominates over the others. Hawking radiation simulation of a black hole showed that the peak of the Hawking radiation lies exactly at the centre of the black hole.

The entropy simulations showed that the entropy of black holes rapidly decreased with time. This was clarified through the Hawking radiation that entropy is carried away to the universe and the second law of thermodynamics is still preserved by the black holes. The entropy variations of rotating black holes whether it is charged or uncharged show that massive black holes are always more stable than the lower mass black holes. Also, supermassive black holes showed a high stability than lower mass black holes as well as heavily charged and large Kerr parameter black holes showed lower stability. This theoretical evidence can be used to verify that the centre of every galaxy has a stable supermassive black hole. These simulations show that the four laws of thermodynamics can be adopted by this model to explain the dynamics of a rotating black hole. The most attractive feature of black holes is that they enlarge our notion of thermodynamics.

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